CHAPTER 7

PRACTICE EXERCISES

SECTION I MULTIPLE CHOICE

1. A mass is attached to a spring on a frictionless, horizontal surface. When it's set into oscillation, its period is T. An equal mass collides head-on with this mass, and the two masses stick together. The oscillation period is now

   (A) T (B) $\sqrt{2}T$ (C) $2T$ (D) $\frac{T}{\sqrt{2}}$ (E) $\frac{T}{2}$

2. A spring-mass system with parameters m and k is oscillating vertically. A second spring-mass system with 3 times the mass is set up beside the first. If the two systems are to oscillate in unison, the spring constant of the second system must be

   (A) $3k$ (B) $\frac{k}{3}$ (C) $\sqrt{3}k$ (D) $\frac{k}{\sqrt{3}}$ (E) $9k$

3. To increase the period of a simple pendulum by a factor of two, you could
   I. double the mass
   II. double the length
   III. quadruple the length

   (A) I only (B) II only (C) III only (D) I and II only (E) I and III only

4. A given pendulum on Earth has a period T. On the Moon, where the acceleration due to gravity is $\frac{1}{6}$ that of Earth, the period will be

   (A) $\frac{1}{6}T$ (B) T (C) $\sqrt{6}T$ (D) $\frac{T}{6}$ (E) $\frac{T}{\sqrt{6}}$

5. A mass M is attached to a spring on a frictionless, horizontal surface and set into oscillation. A smaller mass sits on top of the first and moves with it without slipping. The static friction force exerted on the smaller mass

   (A) is equal to $\mu_s N$ throughout the motion
   (B) reaches a maximum value as the masses reach their maximum speed
   (C) reaches a maximum value as the masses reach their minimum speed
   (D) reaches a maximum value at a point where the speed is between its maximum and minimum values
   (E) remains constant but is less than $\mu_s N$

6. A mass m is attached to a spring with constant k, hung vertically, and allowed to come to rest. Supporting the mass from below so that the spring cannot stretch further, a student adds a second mass $2m$ to the first mass. The support is then removed, and the system begins to execute vertical oscillations with amplitude

   (A) $\frac{mg}{k}$ (B) $2\frac{mg}{k}$ (C) $3\frac{mg}{k}$ (D) $\frac{mg}{2k}$ (E) $\frac{mg}{3k}$

CHAPTER 7

PRACTICE EXERCISES

SECTION II FREE RESPONSE

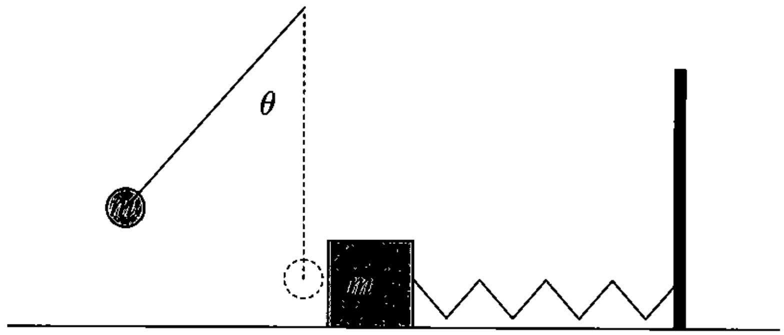

Figure 6

1. A mass m is attached to a light string of length L, making a simple pendulum. It is displaced an angle θ from the vertical and released at $t = 0$. Directly below the pivot of the pendulum is a stationary second mass m equal to the first, attached to a spring of constant k on a frictionless, horizontal surface. When the first mass collides with the stationary mass, the first mass detaches from the string and sticks to the second mass.
   (a) At what time will the spring first reach its maximum compression?
   (b) Find the amplitude of the spring oscillations.

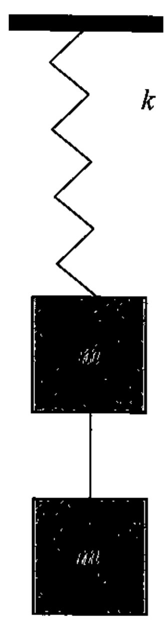

Figure 7

2. Two equal masses m connected by a light string are currently at rest. One of the masses is connected by a spring with constant k to a fixed point directly above it. At $t = 0$, the string is cut, and the mass connected to the spring begins to oscillate.
   (a) Determine the period of the oscillations.
   (b) Determine the amplitude of the oscillations.
   (c) Determine the maximum speed of the mass.
   (d) How much is the spring stretched when the mass is moving at half its maximum speed?

Answers and Explanations

MULTIPLE CHOICE

1. The answer is B. Since $T \propto \sqrt{m}$, it follows that doubling m increases the period by $\sqrt{2}$.

2. The answer is A. Oscillating vertically doesn't affect the period. Since $T \propto \sqrt{\frac{m}{k}}$, if the mass is tripled, then k must be tripled to keep the same period—which is what's required for unison oscillations.

3. The answer is C. The period is independent of the mass and $T \propto \sqrt{L}$, so increasing L by a factor of 4 will double the period.

4. The answer is C. Since $T \propto \frac{1}{\sqrt{g}}$, making g smaller by a factor of $\frac{1}{6}$ will increase the period by a factor of $\sqrt{6}$.

5. The answer is C. The static friction force must behave qualitatively like the spring force on the larger mass. Otherwise, the small mass wouldn't oscillate. This force reaches a maximum when the displacement is at its largest value, and here the speed is 0.

6. The answer is B. The equilibrium point before the second mass is added has a spring stretch given by

$$kx_0 = mg \Rightarrow x_0 = \frac{mg}{k}$$

When the second mass is added and released from rest, it will oscillate about a new equilibrium point determined by

$$kx_0' = 3mg \Rightarrow x_0' = \frac{3mg}{k}$$

The difference in the two equilibrium positions is the original displacement of the system with respect to the new equilibrium point. This is the amplitude since there is no KE here.

$$A = x_0' - x_0 = \frac{2mg}{k}$$

FREE RESPONSE

1. (a) The total time will be $t = \frac{1}{4}T_{\text{pend}} + \frac{1}{4}T_{\text{spring}}$. For the pendulum, you have

$$T_{\text{pend}} = 2\pi\sqrt{\frac{L}{g}}$$

For the spring, $2m$ is oscillating, so you have

$$T_{\text{spring}} = 2\pi\sqrt{\frac{2m}{k}}$$

This means that $t = \frac{\pi}{2}\left(\sqrt{\frac{L}{g}} + \sqrt{\frac{2m}{k}}\right)$.

(b) The maximum compression of the spring will correspond to the amplitude of the oscillations. You can use energy conservation to find the speed of the pendulum mass just before collision.

$$E_i = E_f$$

$$mg(L - L\cos\theta) = \frac{1}{2}mv^2$$

$$v = \sqrt{2gL(1 - \cos\theta)}$$

Next, apply momentum conservation to find the speed of the masses just after collision.

$$P_i = P_f$$

$$mv = 2mv$$

$$V = \frac{1}{2}v = \frac{1}{2}\sqrt{2gL(1 - \cos\theta)}$$

Finally, use energy conservation to determine the maximum compression.

$$E_i = E_f$$

$$\frac{1}{2}(2m)V^2 = \frac{1}{2}kA^2$$

$$A = \sqrt{\frac{2m}{k}}V = \frac{mgL(1 - \cos\theta)}{k}$$

2. (a) Gravity doesn't change the period. Since one mass is oscillating, you have

$$T = 2\pi\sqrt{\frac{m}{k}}$$

(b) Initially the two masses are at rest, so the spring force will just equal the force of gravity.

$$kx_0 = 2mg$$

$$x_0 = \frac{2mg}{k} \qquad x_0 \text{ is initial equilibrium point}$$

After the string is cut, the single mass will oscillate about a new equilibrium point determined by m alone.

$$x_0' = \frac{mg}{k} \quad x_0' \text{ is equilibrium point of single mass}$$

Since no initial speed is imparted to the mass when the string is cut, the difference in the two equilibrium positions will be the amplitude of the oscillation. When the mass returns to x_0, it will be instantaneously at rest again.

$$A = x_0 - x_0' = \frac{mg}{k}$$

(c) Use energy conservation. When you measure spring displacements from the new equilibrium point, gravity can be ignored. The initial energy is then all spring potential energy.

$$E_0 = \frac{1}{2}kA^2 = \frac{1}{2}\frac{(mg)^2}{k}$$

The maximum speed occurs when all the energy is kinetic.

$$\frac{1}{2}mv_{\max}^2 = E_0 = \frac{1}{2}\frac{(mg)^2}{k} \Rightarrow v_{\max} = g\sqrt{\frac{m}{k}}$$

(d) Applying energy conservation using the point where the mass has $\frac{1}{2}$ the maximum speed gives

$$E_0 = E_f$$

$$\frac{1}{2}\frac{(mg)^2}{k} = \frac{1}{2}m\left(\frac{v_{\max}}{2}\right)^2 + \frac{1}{2}kx^2$$

$$\frac{(mg)^2}{k} = m\left(\frac{g\sqrt{\frac{m}{k}}}{2}\right)^2 + kx^2 \Rightarrow x = \frac{1}{\sqrt{2}}\frac{mg}{k}$$